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A DEPOSIT VELOCITY EQUATION FOR OPEN CHANNELS AND PIPES

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Abstract: A simple empirical deposit velocity equation is presented for practical application in the design of pipelines and channels. This model is not seen to represent an advance beyond current theory, but is instead presented as a practical tool for engineers and designers. The model is empirically calibrated with the following data from the literature: Three sets of tailings open channel flume transport data containing 78 measured points, one set of synthetic non-Newtonian slurry flume data containing 95 measured points, and three sets of relevant Newtonian pipe experimental data containing 51 points. These calibrating data sets include turbulent, laminar, homogeneous and heterogeneous slurry behavior. The model has achieved a superior fit to these 223 points of data when compared to a selection of previously published models that remain in common use. The model is presented here for others to test and validate against other data.

KEY WORDS: Deposit velocity, tailings, slurry, transport, channel flow.

NOTATION

C_V	Volumetric solids concentration (as a fraction)
d_{50}	Median particle diameter (m)
D	Pipe internal diameter (m), or equivalent diameter (= 4 R_H)
g	Gravitational acceleration (m/s^2)
R_H	Hydraulic radius of the channel (m)
η^*	Applicable viscosity of the carrier fluid (Pa.s)
η_{inh}	Inherent viscosity of the carrier fluid in a concentrated slurry (Pa.s)
η_B	Bingham plastic viscosity of the slurry, with the Bingham model tangent imposed at a shear rate of at least 400 s^{-1}
ρ_s	Density of the solids (kg/m^3)
ρ_w	Density of the decant fluid (kg/m^3)
τ_y	Yield stress of the slurry mixture (Pa)

1. INTRODUCTION

With the advent of pipeline transport of slurries and other solid/liquid mixtures during the 1950s, it quickly became apparent that deposition of particles in the pipeline was a major operational risk, and this in turn became a major design consideration for subsequent pipeline projects. In the years that followed, a significant amount of effort was devoted to the investigation of the deposit velocity for such slurries and mixtures. Durand (1953) presented an equation for predicting this deposit velocity, which is still widely in use today, albeit with modification in some cases, such as with the improved empirical correlation presented by Wilson and Judge (1976). Other workers have since presented other equations for the transport velocity, with notable entrants being Wasp et al (1977), Thomas (1979) and Oroskar and Turian (1980). It is acknowledged that the state of the art of modeling of pipe flow and deposit velocity has advanced considerably beyond the models that are presented here, but these models are selected for their relative ease of application from a practical design standpoint.

It has also been recognized that these pipeline transport velocity models can provide similar value in the design of open channel slurry transport infrastructure, such as in flumes and launders (Fitton 2007).

Whilst some published deposition velocity models do perform very well under certain conditions, it is found that very few of them can be successfully applied to a very wide range of slurry transport situations. Furthermore, those that can be applied universally still leave room for improvement, with some scatter in predictions compared to experimental data.

The intention of this work is to present a simple transport velocity model that can be universally applied in all slurry flow scenarios, irrespective of whether the slurry is homogeneous (uniformly mixed, non-segregating), heterogeneous (segregating, or sufficiently dilute to have varying density with depth), turbulent or laminar.

While this goal may seem ambitious, it is noted that at least two transport velocity models already exist in the literature that arguably meet this objective already; the Wasp et al (1977) model and the Oroskar and Turian (1980) model. Both of these models have already been found to be reasonably accurate in predicting transport velocities for laminar, turbulent homogeneous and turbulent heterogeneous slurries (Fitton 2007). It is therefore desirable that the new deposit velocity model should make more accurate predictions than those two models to be of practical value to designers.

2. CALIBRATING DATA

The data that has been used to calibrate this new transport velocity model has been collected in seven separate experimental campaigns. Each campaign focused on identifying the minimum transport flow conditions for a given slurry or solid/fluid mixture, in which the deposition velocity was defined as the velocity at which the onset of particle deposition was occurring. The author conducted four of the campaigns, which featured open channel flumes, with three of these four flumes being set up at mine sites on a full scale or pilot scale. A photograph of one of these flumes is presented as Figure 1. For each measured data point in those four open channel campaigns, a slurry (or particle/fluid mixture) flowed along a sloping open channel flume at a nominal flow rate, and the slope of the flume was periodically adjusted to determine which slope would induce deposition of particles on the channel bed. The primary method of detecting deposited particles was by feeling the bed of the channel with the fingertips after maintaining uniform steady flow conditions for about 10 minutes.

Figure 1. Photograph of the experimental open channel flume used at Chuquicamata, Chile

Figure 1 shows a tilting flume used at a pilot scale test facility at the Chuquicamata copper mine in Chile. The system included an agitated tank and a recirculating pump, whilst the flume angle was adjusted using the overhead chain blocks.

The experimental data gathered from these open channel flume experiments was initially used to develop a beach slope prediction model for tailings deposits, which has since been practically applied in industry for the design of tailings storage facilities. The data has also been used to develop models for the design of long distance open channel flumes for the transport of tailings slurries.

The remaining three experimental campaigns were carried out in horizontal pipes running full. It is noted that the seven sets of data contain a wide range of slurry flow conditions that feature laminar, turbulent, concentrated, dilute, homogeneous, heterogeneous, open channel and pipe flow regimes, with equivalent pipe diameters ranging from 10 mm to 300 mm. All seven sets of data have been previously published.

Table 1 summarizes some of the key aspects of the seven data sets, and provides the references for the original publication that presented each data set. It is noted that some description and detail of those experimental campaigns can be found in those references.

Table 1. Calibrating data summary table

Data set	Description	No. Points	SG solids	SG fluid	d_{50} micron	D mm	C_w % w/w	$\square_B$ mPa.s	Ref
Sunrise Dam	Gold tailings in brine, open channel	41	2.8	1.15	16	59.1-222.5	25.8-66.8	2.21-24.0	Fitton 2007
Peak (Cobar)	Gold tailings, open channel	8	2.8	1	7.8	77.0-179.7	53.3-57.7	10.1-19.6	Fitton 2007
Chuquicamata	Copper tailings, open channel	29	2.75	1.01	80	88.9-232.2	56.1-68.0	7.90-53.0	Pirouz et al 2013
RMIT Small Flume	Crushed glass in CMC solution, open channel	95	2.64	1	335-1000	29.3-60.5	0.004-13.2	1.00-22.7	Fitton 2007
Thomas silica/water	Silica sand in water, pipe flow	10	2.65	1	130-1200	9.4-105.0	26.5	0.80-0.89	Thomas 1979
Thomas sand/fluid	Sand in brine and other Newtonian fluids, pipe flow	17	2.65-7.48	0.77-1.35	17-900	18.9-105.0	21.1-57.0	0.80-56.0	Thomas 1979
Schrieck silica/water	Silica sand in water, pipe flow	23	2.66	1	180	52.2-105.0	24.7-59.9	1.13	Schrieck et al 1973

It is acknowledged that the selected validating data is limited and not exhaustive, but it is seen as a suitably diverse data set for the intent of this work.

3. THE NEW MODEL

The new deposit velocity model is an empirically calibrated equation, which contains many of the same terms as some other previously published deposit velocity models, particularly that of Wasp et al (1977). The new model is presented as follows:

$$V_d = 1.48\, C_v^{0.19} (d_{50}/D)^{0.16} (2gD(\rho_s/\rho_w - 1))^{0.50} \eta^{*-0.12} \qquad (1)$$

Like all other deposit velocity models, this new model can also be applied to open channels by substituting D for $4R_H$, where R_H is the hydraulic radius of the channel.

The applicable viscosity of the carrier fluid will be one of the following three values:

- For high concentration slurries with wide particle size distributions (as is commonly the case with tailings slurries), the **inherent viscosity** should be used.
- For slurries with a clearly identifiable and separable non-Newtonian carrier fluid, the **Bingham plastic viscosity** measured at a tangent of at least 400 s^{-1} should be used.

- For dilute slurries with coarse particles and Newtonian carrier fluids, the viscosity of the carrier fluid should be used.

The inherent viscosity was proposed by Thomas (2010). It was developed for high concentration slurries (>12% w/w) with a wide particle size distribution, as is commonly the case with tailings slurries. In such slurries the finer particles form part of the carrier fluid, but due to the difficulty in defining which particles do form the carrier fluid it is not possible to measure the rheology of the carrier fluid alone. Almost all the previously published deposit velocity models have been developed based on discrete particles in a known carrier fluid such as water, a viscous Newtonian fluid, or a fine clay slurry. The problem is that most practical slurries of interest in the mining industry have a wide particle size and possess both non-Newtonian properties and settling tendencies. It is possible to measure the rheology of the slurry but to apply the deposition velocity models the properties of the carrier fluid portion are required. Thomas therefore argued that the rheological testing of the whole slurry would result in there being a degree of inter-particle mechanical contact that would cause the measured viscosity to be larger than the viscosity of the carrier fluid portion. He presented an equation for correcting the measured viscosity to obtain the actual viscosity of the carrier fluid, which he referred to as the "inherent viscosity". The Thomas (2010) equation for the inherent viscosity is as follows:

$$\eta_{inh} = \eta_B / e^{2.7(Cv / (1-Cv))} \quad (2)$$

4. TESTING OF PUBLISHED MODELS

5.1 SELECTED MODELS

A number of published transport velocity models were tested in the work of Fitton (2007). From that work, a selection of the best performing models has been tested here, to compare with the current new model. These are:

Wasp et al (1977): $V_d = 3.8\, C_v^{0.25}(d_{50}/D)^{1/6}(2gD(\rho_s/\rho_w-1))^{0.50}$ (3)

Thomas (1979): $V_d = 9(g\eta_B(\rho_s/\rho_w-1)/\rho_w)^{0.37}(4R_H\rho_w/\eta_B)^{0.11}$ (4)

Oroskar and Turian (1980): $V_d = (gD(\rho_s/\rho_w-1))^{0.50}1.85C_v^{0.1536}(1-C_v)^{0.3564}\ldots$

$\ldots(d_{50}/D)^{-0.378}\,(D\rho_w(gd_{50}(\rho_s/\rho_w-1))^{0.5}/\mu)^{0.09}0.97^{0.30}$ (5)

It is noted that some of these models will be applied against data that they were not intended to describe. However, in the interests of the overall aim of this work (to produce a transport velocity model that applies to all flow regimes), it is considered appropriate that these existing models should be tested in the same capacity, as it may well be found that some of them do perform well against a data set of such assorted flow regimes.

Fitton (2007) produced a model that uses the average velocity as an input parameter, which, under the circumstances of this assessment, gives it an unfair advantage over the other models. It is essentially using the answer to make its prediction. For this reason, the Fitton (2007) model has not been tested here. The Wilson and Judge (1975) modification of the Durand (1953) model is useful for moderately settling slurries. However it is generally not applicable to the near homogeneous slurries of interest here. It can be noted that Thomas (1979) developed a model for predicting the deposit velocity for very fine particle slurries and provided a method of combining this model with the model of Wilson and Judge. The Thomas (1979) predictions here are based on this combined approach.

5.2 ASSESSMENT OF MODEL PERFORMANCE

In this work an assessment of the predictive accuracy of the new model using an independent data set has not been undertaken, since any available data of relevance has instead been used for the calibration of the model. However, some effort has been made to gauge its goodness of fit in comparison to the four selected models from the literature, by testing the performance of those published models against the same set of calibrating data that the new model has been fitted to. The absolute error has been selected as the assessment criterion for the accuracy of the prediction. The absolute error for each point is equal to the difference between the predicted transport velocity and the calculated average velocity. A low average absolute error indicates a good fit to that data set.

A fit plot for the new model is presented as Figure 2. In that figure, the observed deposit velocity is plotted against the horizontal axis, whilst the predicted deposit velocity is plotted against the vertical axis. The diagonal line running through the origin is the ideal fit line. A perfect prediction will fall on this line.

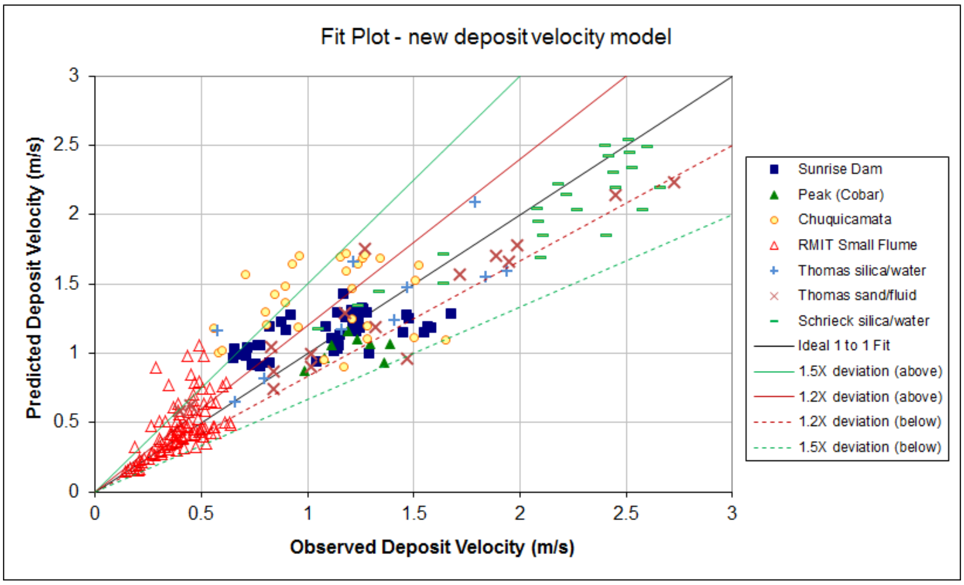

Figure 2. Fit Plot for the new deposit velocity model

A summary of the average absolute errors for each of the tested models (including the new model) is presented in Table 2. Also presented in that table is average absolute error for the three tailings data sets only.

Table 2. Comparison of model fit to data

Deposit Velocity Model	**Average Absolute Error (m/s)**		**Sum of errors (m/s)**
	Overall	**Tailings only**	
New model	0.188	0.274	0.462
Wasp et al 1977	0.217	0.333	0.550
Thomas 1979	0.599	0.288	0.887
Oroskar and Turian 1980	0.287	0.343	0.630

From Table 2 it can be seen that the new model fits its calibrating data better than any of the tested published models, achieving an average absolute error of 0.188 m/s. The new model has also fit the tailings data more accurately than the other tested models, with an average absolute error of 0.274 m/s. For the overall data, the closest performer from the literature was the Wasp et al. (1977) model, which achieved an average absolute error of 0.217 m/s, about 15% higher than the new model. For the tailings data only, the closest performer from the literature was the Thomas (1979) model, which achieved an average absolute error of 0.288 m/s, about 5% higher than the new model.

5. DISCUSSION

The new model has been found to fit the calibrating data with greater accuracy than any of the tested published models by a factor of 15% or more, though it is noted that this is not a definitive assessment of the predictive accuracy of the new model, since the data that has been used to assess this accuracy is the same data that was used to calibrate the model. It is therefore hoped that the model can be validated against some independent data, so that this advantage is removed.

It is noted that the validity of this new model has not been tested for laminar flow in large pipes. Thomas (1979) observed that laminar flows could be sustained for long distances in small diameter pipes, but particles from the same slurry at the same velocity were found to settle in large diameter pipes due to the required pressure loss gradient for transport being higher. It is therefore noted that this new equation may not apply for laminar flows in large pipes.

It is acknowledged that some of the tested published models were presented for slurries flowing in a particular regime, such as turbulent homogeneous slurries, or turbulent heterogeneous slurries, so the application of such models in this work to cover all cases may place the model in an situation that it is not intended to cover, and one which it may not have been previously validated for. In spite of this, it is noted that some of these published models do appear to apply quite well to the full range of the calibrating data sets, even though the experimental data contains many points in the laminar range, and many points in highly dilute heterogeneous regimes (particularly with respect to the RMIT data). In particular, the Wasp et al. (1977) model and the Oroskar and Turian (1980) model can be commended for their accuracy across this data.

6. CONCLUSIONS

A new empirical deposit velocity model has been presented, not as an advance on the current state of theoretical understanding of deposition modelling, but for practical design applications. The model has been empirically calibrated with three sets of tailings transport data containing 78 measured points, and four other sets of relevant Newtonian and non-Newtonian experimental data containing 145 points. These calibrating data sets include turbulent, laminar, homogeneous and heterogeneous slurry behavior. The model has achieved a superior fit to these 223 points of data when compared to a selection of previously published models in common use. The model is presented here for others to test and validate with other data.

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REFERENCES

1. Durand, R. (1953) Basic relationships of the transportation of solids in pipes - experimental research, paper presented to Minnesota International Hydraulics Convention, Minnesota, USA.
2. Fitton, T.G. (2007) Tailings beach slope prediction, PhD thesis, RMIT University, 2007. (Published as a book by VDM Verlag Germany in 2010, ISBN 9783639223729)
3. Oroskar, A.R. and Turian, R.M. (1980) The critical velocity in pipeline flow of slurries', AIChE J., vol. 26, pp. 550-8.
4. Schrieck, W., Smith, L.G., Haas, D.B., and Husband, W.H.W (1973) Experimental studies on the transport of two different sands in water in 2, 4, 6, 8, 10 and 12 inch pipes, Experimental Studies on Solids Pipelining of Canadian Commodities for the Canadian Transport Commission and the Transportation Development Agency, Report VII
5. Pirouz B., Seddon, K.D., Pavissich, C., Williams, M.P.A., Echevarria, J. (2013) Flow Through Tilt Flume Testing for Beach Slope Evaluation at Chuquicamata Mine Codelco, Chile, 16th International Seminar on Paste and Thickened Tailings, Belo Horizonte, Brazil, June 2013
6. Thomas, A.D. (1979) Predicting the deposit velocity for horizontal turbulent pipe flow of slurries, Int. J. Multiphase Flow, Vol.5, pp113-129
7. Thomas, A.D. (2010) Method of determining the inherent viscosity of a slurry and other rheological trends as illustrated by a data bank of over 200 different slurries. Hydrotransport 18 Conf., Rio de Janeiro, Brazil, Sept 22-24
8. Wasp, EJ, Kenny, JP & Gandhi, RL 1977, Solid-liquid flow slurry pipeline transportation, First edn, Trans Tech Publications, Germany.
9. Wilson, K.C. and Judge, D.G. (1976) New techniques for the scale-up of pilot plant results to coal slurry pipelines, Univ. of Pennsylvania, Proc. Int. Symp. On Freight Pipleines, Washington, DC, pp 1-29, December 1976.